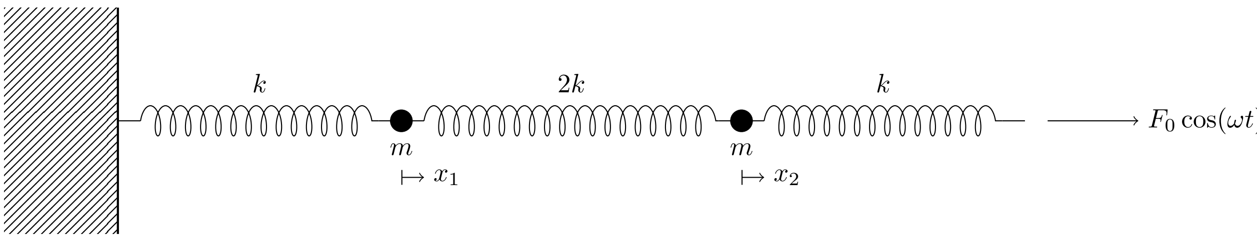
PHYS 2601: Classical and Quantum Waves Practice Midterm 2

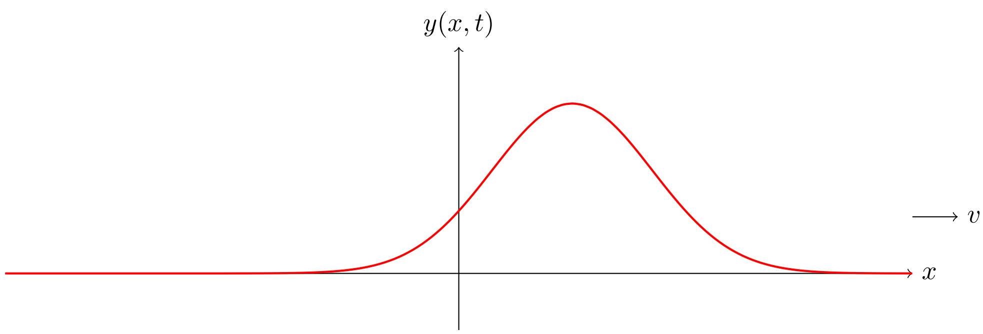
Professor James McIver

Problem 1 (30 points)



Suppose we have a driven coupled oscillator system as modeled above. The left and right springs have spring constant k while the middle spring has spring constant $2k$. The masses are identical. At the time $t = 0$ a driving force is applied on the rightmost spring, given by $F(t) = F_0 \cos \omega t$.
a) Write the differential equation of motion for x_1 (make sure the signs are correct). (5 points)
b) Write the differential equation of motion for x_2 (make sure the signs are correct). (5 points)
c) Using the results from parts **a** and **b**, write the uncoupled equations of motion for $q_1 = x_2 + x_1$ and $q_2 = x_2 - x_1$. What are the resonant angular frequencies ω_1 and ω_2 for q_1 and q_2? (5 points)
d) Suppose our system approaches a steady-state solution. Solve for $q_1(t)$ and $q_2(t)$ to get solutions for $x_1(t)$ and $x_2(t)$. (10 points)
e) Describe the motion of the masses as the driving angular frequency ω approaches ω_1 and ω_2. (5 points)

Problem 2 (20 points)



Suppose we have a very long string with linear mass density μ. A Gaussian-shaped wave propagates to the right with velocity v, described as follows:

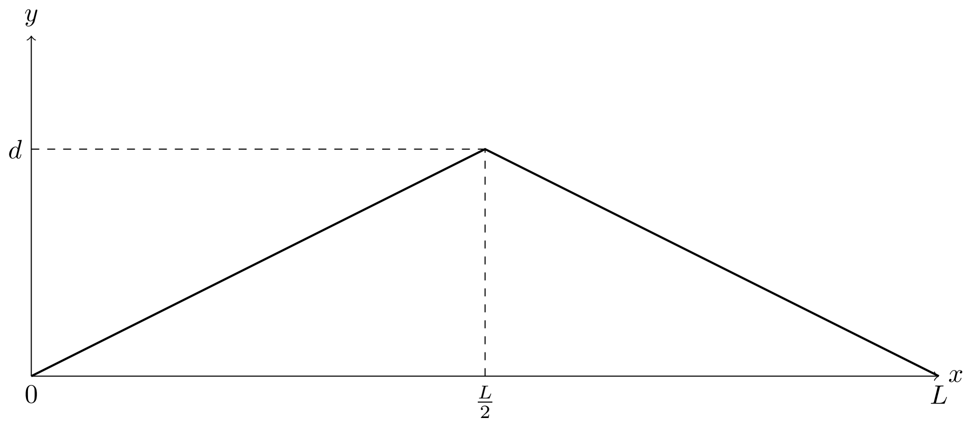
$$y(x,t) = A\bar{e}^{(x-vt)^2/2\sigma^2}$$

a) Verify that the Gaussian function satisfies the 1-dimensional wave equation. (5 points)
b) Solve for the energy of the entire Gaussian wave. You may use the following integral:

$$\int_{-\infty}^{\infty} x^2 e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$$

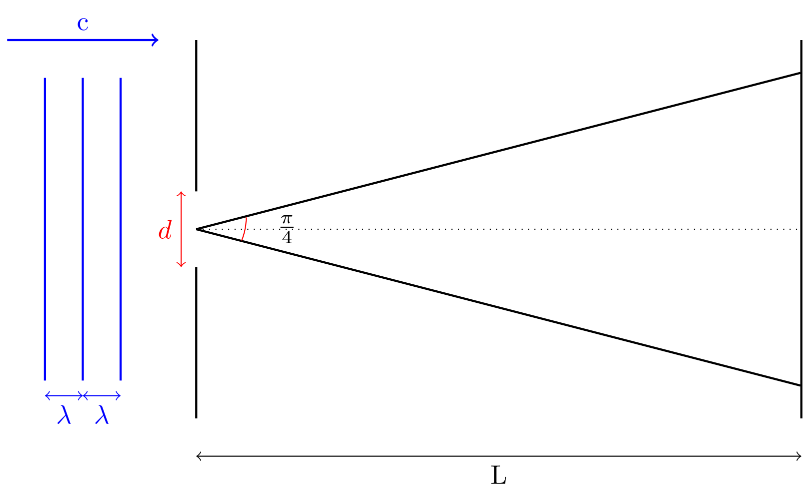
(10 points)
c) Solve for the magnitude of the maximal transverse velocity of the wave. (5 points)

Problem 3 (25 points)



Suppose we have a string with length L, linear mass density μ, under constant tension T with its ends at $x = 0$ and $x = L$ fixed. The midpoint of the string is displaced by a distance d and released at time $t = 0$.
a) $f(x)$ models the shape of the string at $t = 0$. Solve for $f(x)$. (5 points)
b) Find the amplitude of the n-th normal mode excited in the string. (10 points)
c) Using your answer from part **b**, for which values of n is the amplitude of the n-th normal mode zero? (5 points)
d) Solve for the energy of the n-th normal mode. (5 points)

Problem 4 (15 points)



Suppose we have monochromatic light of wavelength λ and frequency ν passing through a slit of width d. A screen is set at a distance $L >> d$ to the right of the slit where we can observe a diffraction pattern.
a) In the diffraction pattern, we observe 6 minima within an angular interval $\frac{\pi}{4}$ as indicated in the diagram. Determine the minimum and maximum values of λ to produce this effect on the screen. (5 points)
b) Suppose λ is the maximal value you found in part **a**. By what factor should we increase the frequency ν to **just** observe 8 minima within the angular interval? (HINT: this is a single value, not an interval) (5 points)
c) Suppose the slit width d is halved. Describe the qualitative changes to the diffraction pattern. (5 points)